

A Lifecycle Estimator for Intergenerational Earnings Mobility

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Life course analysis: data and methodological challenges
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Introduction

- Estimating income mobility is challenging and most work has to rely on imperfect data.
- Ideally, require lifetime income for two generations. For example, to estimate "intergenerational elasticity of income",

$$y_{s,i}^* = \alpha + \beta y_{f,i}^* + \varepsilon_i \quad (1)$$

- But usually researchers do not observe lifetime income:
 - Using annual income might lead to large biases. Current standard in literature: aim to measure incomes around "midlife" (Haider and Solon, 2006) 
 - Measuring incomes around midlife reduces lifecycle bias, but does not eliminate it (Nyblom and Stuhler, 2017)
 - Moreover, in many settings we observe incomes only at early ages; for example, in estimation of mobility trends for recent cohorts
- Measurement matters: Reliability of comparisons across space and time also affects analysis of causal mechanisms 

Mello, Nybom and Stuhler (2021)

Propose an estimator of intergenerational income mobility that eliminates lifecycle bias and that could be applied in common data settings faced by practitioners; in particular, in short panels of young cohorts.

- Build on a smaller strand of the literature that proposes to “model the income process” by estimating complete income profiles based on partial profiles and observable characteristics (Creedy, 1987; Hertz, 2007; Vogel, 2007).
- Propose an improved “lifecycle estimator” and test its performance in different settings (Swedish data, PSID and simulated data).
- Apply the lifecycle estimator to revisit the question of recent trends in intergenerational mobility in the U.S. and Sweden.
- Note: focus on the IGE and on income of offspring.

Data

Swedish administrative registers (Statistic Sweden and IFAU).

- Individuals' income trajectory and rich characteristics (gross labor earnings, education, occupation, region of birth, cognitive and non-cognitive skills, parental links)
- Main Sample: 201,063 sons born from 1952-1960
 - Individual labor earnings for sons aged 25-58 ("lifetime income") and fathers aged 41-57; restrictions on parental age
- Trends Sample: 1,955,368 sons born in 1950-1989

Panel Study of Income Dynamics (PSID)

- US household survey with intergenerational links, waves released between 1968 and 2017
- Parental income: family income over child age 15-17 (similar to Lee and Solon 2009 and Chetty et al. 2014)
- Main sample: approx. 1,000 sons and daughters born between 1951 and 1960
- Trends sample: approx. 4,000 sons and daughters born in 1950-1989

Intergenerational Perspective on Income Process

As a reference point, consider the HIP model by Guvenen (2009). Let log income for individual i with experience h at time t be given by:

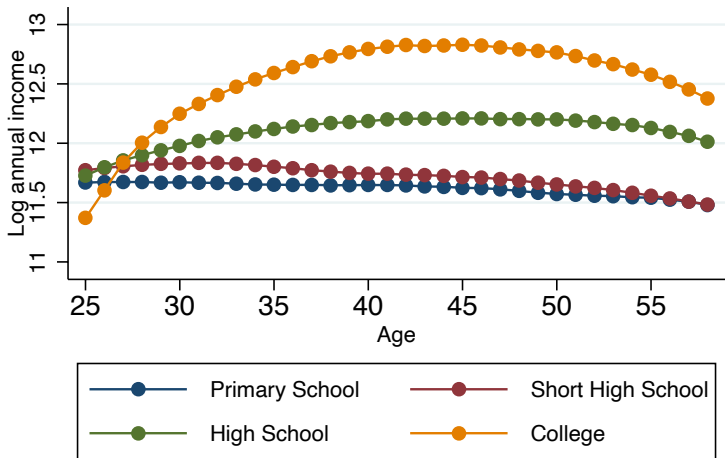
$$Y_{h,t}^i = g(\theta_t, X_{h,t}^i) + \alpha^i + \beta^i h + z_{h,t}^i + \phi_t \varepsilon_{h,t}^i$$

The income process has three key important components for our setting:

- $g(\theta_t, X_{h,t}^i)$, the income growth explained by observed characteristics;
- ε_{ht}^i , the transitory noise;
- $\alpha^i + \beta^i h$, the unexplained income growth, that nevertheless may correlate within families.

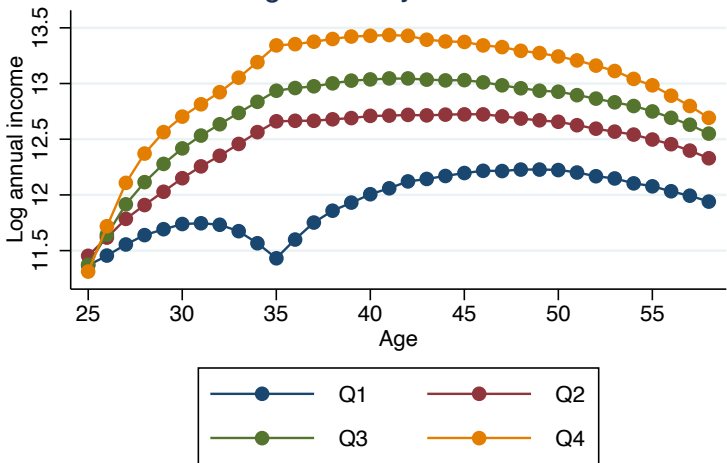
Components of the Income Process

A: Income by Education



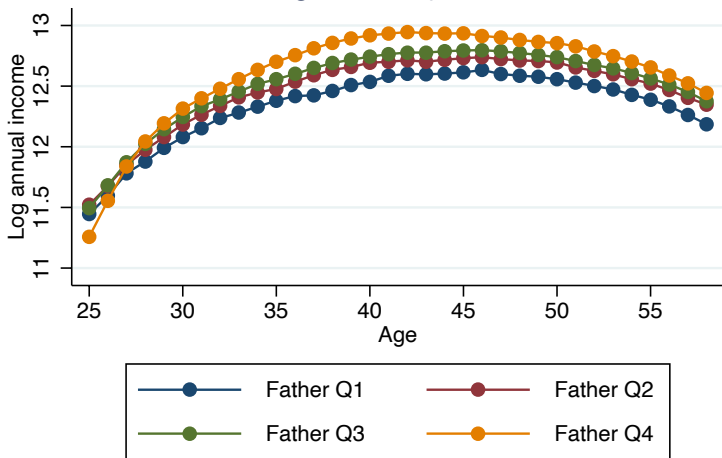
Components of the Income Process

B: College Grad by Own Annual Inc



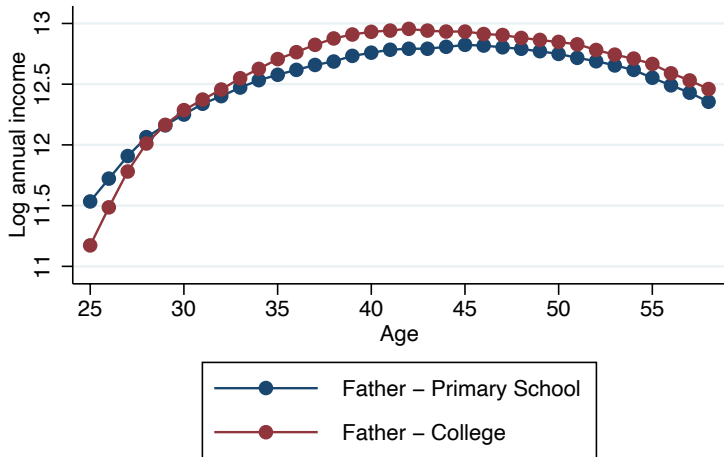
Components of the Income Process

C: College Grad by Fathers' Inc



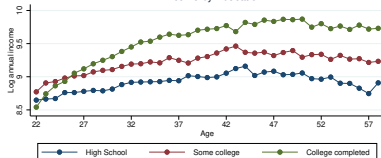
Components of the Income Process

D: College Grad of High-inc Fathers by Educ

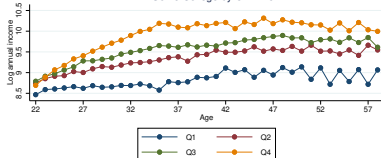


Components of Income Process: PSID

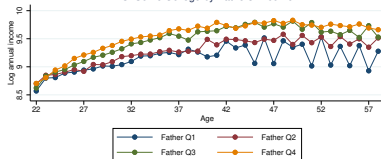
A: Income by Education



B: Some College by Own Inc



C: Some College by Fathers' Income



D: Some College of High-inc Fathers' by Education

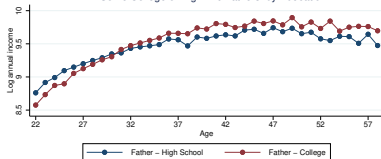


Table 2: Earnings Growth Heterogeneity by Father's Income, Swedish data

	(1)	(2)	(3)	(4)	(5)	(6)
Log (Father's Income)/100						
x Age 25-30	13.3857*** (0.2521)	3.6746*** (0.2273)	3.4935*** (0.2626)	5.8631*** (0.2410)	1.4998*** (0.2253)	1.6059*** (0.2621)
x Age 30-35	6.8129*** (0.2016)	1.5028*** (0.2036)	0.8233*** (0.2306)	2.5025*** (0.2040)	0.4841* (0.2048)	0.1163 (0.2337)
x Age 35-40	3.1877*** (0.1927)	0.1389 (0.1976)	0.0653 (0.2263)	0.9817*** (0.2263)	0.0508 (0.2269)	0.0278 (0.2554)
x Age 40-45	0.7383*** (0.1810)	-0.4760* (0.1884)	-0.2721 (0.2160)	-0.0421 (0.2087)	-0.5428* (0.2115)	-0.4158 (0.2445)
x Age 45-50	-0.5426** (0.1771)	-0.1232 (0.1829)	-0.2768 (0.2107)	-0.2270 (0.1903)	-0.0125 (0.1926)	-0.0087 (0.2244)
x Age 50-55	-2.4630*** (0.1742)	-0.8726*** (0.1790)	-0.6699** (0.2063)	-1.3924*** (0.1810)	-0.5916** (0.1807)	-0.4629* (0.2097)
Education x Age		X	X		X	X
Occupation x Age				X	X	X
Skill scores x Age			X			X
Demographics x Age						X
N	950263	950125	744286	616941	616895	484297
R-sq	0.053	0.117	0.122	0.132	0.186	0.192

Existing Correction Methods

Modelling the Income Process

Strategy I: Observable Heterogeneity with Individual Fixed Effects (Hertz 2007, Vogel 2007)

- 1 Estimate income profiles with growth rates depending on observables and individual fixed effects

$$y_{ict} = \alpha_i + f(\text{Age}_{ict})\beta + g(\text{Age}_{ict}, Z_{ic})\gamma + \alpha_t + \varepsilon_{ict}, \quad (2)$$

- $f(\text{Age}_{ict})$ is a polynomial in age
 - $g(\text{Age}_{ict}, Z_{ic})$ is a flexible interaction of age with a vector of individual observables (such as education)
- 2 Predict income at one particular age (Hertz, 2007) or predict and aggregate over lifecycle (Vogel, 2007).

Strategy II: Extrapolate from income levels to income slopes (Creedy, 1988)

Existing Correction Methods

Modelling the Income Process

Performance:

- Accounting for individual FE and income growth that depends on observables (e.g. education/occupation) reduces lifecycle bias as compared to directly using annual incomes.
- Yet, income growth varies even within education/occupation/other observables in a way that is systematically correlated with parental background.
- Estimated fixed effects - and lifetime income - therefore depend on age range included in the first step estimation. Figure
- Then, when observing early ages, we understate lifetime income of those with low initial incomes and steep profiles.
- Biased estimation of mobility trends, in particular when using different age windows for different cohorts. Hertz Trends

Insight: observable heterogeneity and fixed effects are not enough to capture effect of parental background on income profiles.

Correction Methods

Modelling the Income Process

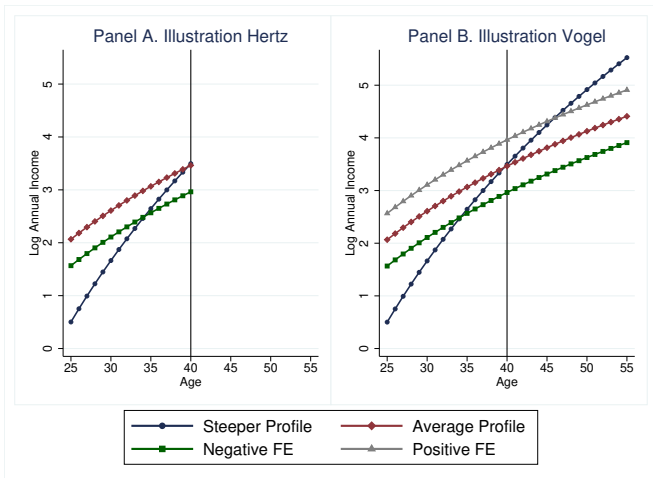
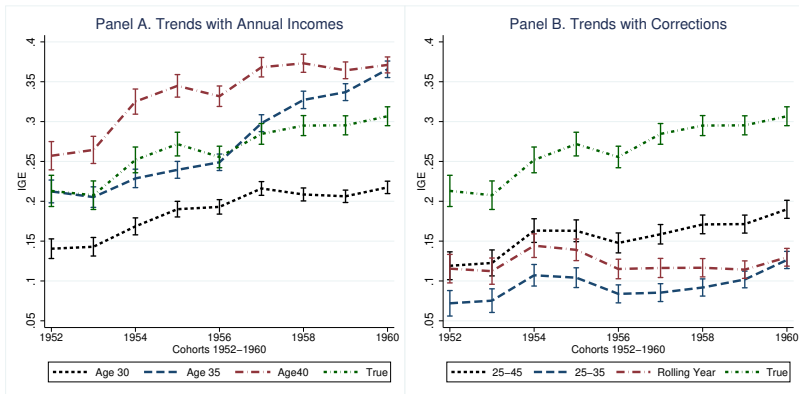


Figure: Illustration of life-cycle bias

Correction Methods

Modelling the Income Process

Consequences for estimation of mobility trends



Lifecycle Estimator

Our Proposal: Lifecycle Estimator

- 1 Estimate lifecycle profiles allowing for individual fixed effects and slopes that vary both with own and parental characteristics. Specifically, estimate:

$$y_{ict} = \alpha_i + f(A_{ict})\beta + g(A_{ict}, Z_{ic})\gamma + h(A_{ict}, Z_{ic}, P_{ic})\delta + \varepsilon_{ict}, \quad (3)$$

- α_i are individual FEs, $f(A_{ict})$ is a polynomial in age; $g(A_{ict}, Z_{ic})$ is an interaction of age with a vector of the individual's own characteristics
 - $h(A_{ict}, Z_{ic}, P_{ic})$ is an interaction of child age and education with parental characteristics P_{ic}
- 2 Predict lifetime income and estimate the IGE.

Lifecycle Estimator

Parental Lifecycle Estimator

- P_{ic} contains parental lifetime income and four indicators for parental education.
- Consider linear or quadratic interaction between age and parental income (see Table 2).

Two-step FE Estimator

- Estimate equation (3) without the $h(A_{ict}, Z_{ic}, P_{ic})$ interaction, to yield estimates of the individual fixed effect $\hat{\alpha}_i$.
- Re-estimate the equation with the $h(A_{ict}, Z_{ic}, \hat{\alpha}_i)$ interaction (linear or quadratic)

Lifecycle Estimator

Testing the performance of the lifecycle estimator

- Birth cohort 1952-1960 in Swedish registers, observe complete profiles (age 25-58)
- We therefore know “true” lifetime income and intergenerational elasticity
- Split each individual into two partial copies and apply lifecycle estimator on partial profiles

Issues:

- Estimation consists of multiple steps, affecting statistical inference.
- Conversion of log to absolute incomes gives rise to so-called *re-transformation problem* ($E[\hat{\varepsilon}_{ict}] = 0$, but $E[\exp(\hat{\varepsilon}_{ict})] > 0$)
- In many applications, child generation is only observed at young age, and incomes at later ages never observed.

Son's Age	N	Direct estimator		Lifecycle estimator				
		Lifetime	Annual	Baseline	Parental (Linear)	Parental (Quadratic)	2-Step (Linear)	2-Step (Quadratic)
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Age $\leq$ 27	375952	0.288*** (0.002) 0.070	0.070*** (0.002) 0.004	0.200*** (0.002) 0.032	0.225*** (0.002) 0.040	0.268*** (0.002) 0.056	0.225*** (0.002) 0.024	0.235*** (0.003) 0.021
Age $\leq$ 30	374376	0.289*** (0.002) 0.072	0.120*** (0.002) 0.010	0.237*** (0.002) 0.050	0.251*** (0.002) 0.055	0.293*** (0.002) 0.075	0.268*** (0.002) 0.043	0.285*** (0.002) 0.040
Age $\leq$ 33	372548	0.289*** (0.002) 0.072	0.186*** (0.002) 0.024	0.246*** (0.002) 0.058	0.252*** (0.002) 0.061	0.290*** (0.002) 0.080	0.272*** (0.002) 0.053	0.285*** (0.002) 0.050
Age $\leq$ 36	370558	0.289*** (0.002) 0.072	0.206*** (0.002) 0.028	0.253*** (0.002) 0.063	0.263*** (0.002) 0.068	0.295*** (0.002) 0.085	0.277*** (0.002) 0.058	0.289*** (0.002) 0.055
Age $\leq$ 40	367404	0.287*** (0.002) 0.073	0.242*** (0.002) 0.034	0.264*** (0.002) 0.067	0.294*** (0.002) 0.083	0.302*** (0.002) 0.087	0.293*** (0.002) 0.062	0.299*** (0.002) 0.061
Age $\leq$ 45	363026	0.285*** (0.002) 0.072	0.300*** (0.003) 0.038	0.274*** (0.002) 0.071	0.307*** (0.002) 0.088	0.295*** (0.002) 0.082	0.300*** (0.002) 0.067	0.297*** (0.002) 0.068

Lifecycle Estimator

Performance in alternative scenarios:

- Mean estimates are fairly insensitive to:
 - the age range available for first step estimation
 - the number of income observations observed for each person [Table](#)
 - the number of individuals in the sample [Table](#)
- The lifecycle estimator also performs well in the PSID; parental/FE quadratic estimators fluctuate around benchmark estimate, without any apparent systematic lifecycle bias. [Table](#)

Trends in Income Mobility

- Study mobility trends in Sweden and the US to:
 - Examine whether previous estimates may have been systematically distorted by lifecycle bias
 - Estimate mobility trends for younger, more recent cohorts
- For recent cohorts, income profiles are necessarily incomplete.
 - Possible "solution": assume shape of age-income profiles remains constant across cohorts (Vogel, 2007; Haider and Solon, 2006).
 - Alternatively, allow steepness of income profiles to vary across cohort groups (separately by education group).

Table 8: Trends in Income Mobility in Sweden (Register data)

Cohort	Direct Estimator		Lifecycle estimator				
	Annual All ages	Annual Ages 25-30	Baseline	Parental (Quadratic)	Parental (Quadratic) Cohort Adj. 1	Parental (Quadratic) Cohort Adj. 2	2-Step (Quadratic)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1950-59	0.230*** (0.002)	0.087*** (0.002)	0.196*** (0.002)	0.196*** (0.002)	0.196*** (0.002)	0.196*** (0.002)	0.190*** (0.002)
1960-69	0.224*** (0.002)	0.137*** (0.002)	0.207*** (0.002)	0.212*** (0.002)	0.212*** (0.002)	0.212*** (0.002)	0.221*** (0.002)
1970-79	0.198*** (0.002)	0.161*** (0.002)	0.197*** (0.002)	0.204*** (0.002)	0.204*** (0.002)	0.204*** (0.002)	0.229*** (0.002)
1980-89	0.162*** (0.002)	0.154*** (0.002)	0.179*** (0.002)	0.191*** (0.002)	0.186*** (0.002)	0.190*** (0.002)	0.222*** (0.002)
R2	0.025	0.017	0.044	0.047	0.046	0.046	0.040
N	39,148,343	9,921,334	1,844,829	1,844,829	1,844,829	1,844,829	1,844,829

Table 9: Trends in Income Mobility in the U.S. (PSID)

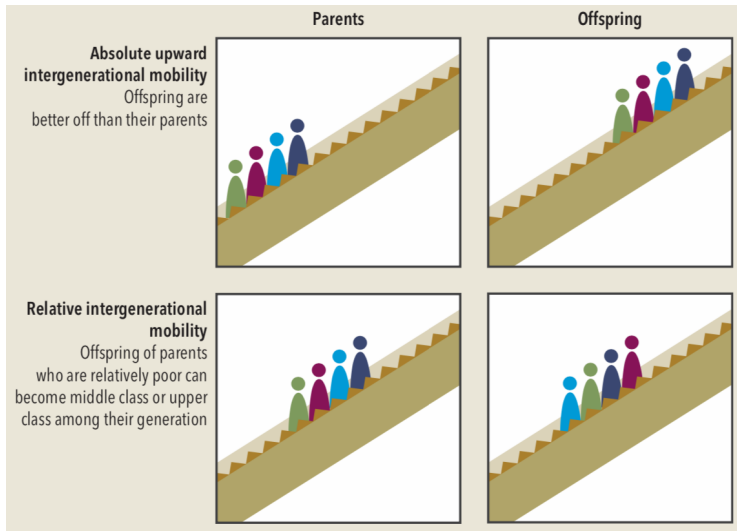
	Direct Estimator		Lifecycle estimator				
	Annual All ages	Annual Ages 25-30	Baseline	Parental (Quadratic)	Parental (Quadratic) Cohort Adj. 1	Parental (Quadratic) Cohort Adj. 2	2-Step (Quadratic)
Cohort	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1950-59	0.3796*** (0.0336)	0.3093*** (0.0394)	0.4328*** (0.0409)	0.4207*** (0.0410)	0.4221*** (0.0410)	0.4229*** (0.0409)	0.4392*** (0.0430)
1960-69	0.3911*** (0.0341)	0.3614*** (0.0369)	0.4392*** (0.0362)	0.4347*** (0.0362)	0.4326*** (0.0360)	0.4330*** (0.0362)	0.4531*** (0.0387)
1970-79	0.4060*** (0.0293)	0.3489*** (0.0334)	0.4588*** (0.0327)	0.4536*** (0.0333)	0.4568*** (0.0333)	0.4561*** (0.0333)	0.4913*** (0.0368)
1980-89	0.3079*** (0.0253)	0.3111*** (0.0273)	0.3714*** (0.0254)	0.4009*** (0.0251)	0.4202*** (0.0256)	0.4130*** (0.0252)	0.4109*** (0.0301)
R2	0.0867	0.0820	0.1476	0.1506	0.1530	0.1533	0.1346
N	59458	17616	4931	4931	4931	4931	4931

Concluding Remarks

- Lifecycle estimator that allows for heterogeneous slopes (by parental or own income) performs well, fluctuating around benchmark without any apparent systematic lifecycle bias.
- Estimator is fairly insensitive to age range available for first step estimation, to the number of income observations available for each individual and to the number of individuals in sample.
- Estimator is attractive for comparative purposes, e.g. for studying mobility across countries and time.
- Estimator leads to slightly different conclusions regarding trends estimation in Sweden and the US.
- Limitations: we have not addressed RHS measurement error or alternative mobility statistics (e.g., rank mobility)

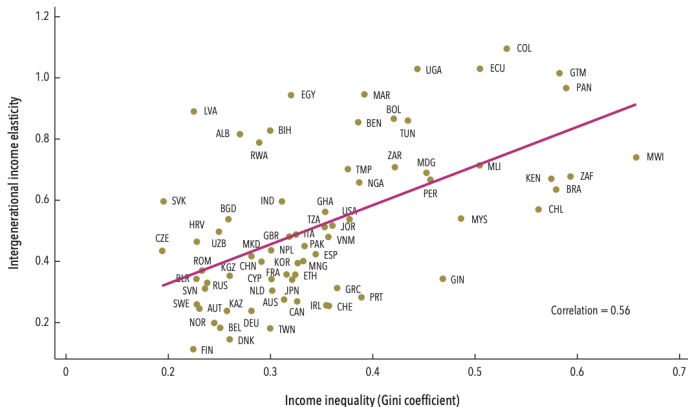
Figures and Tables

Introduction

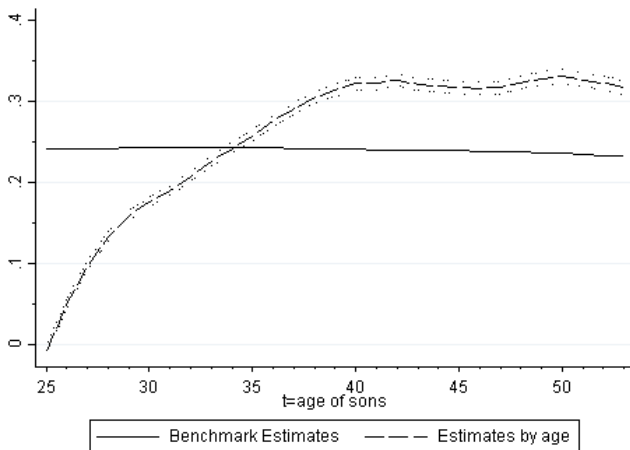


Introduction

FIGURE 4.2 Higher relative IGM in income is associated with lower income inequality



Lifecycle Bias



- Focus on Left Hand Side (LHS) measurement error because we do not observe the complete lifecycle of young cohorts and would like to measure mobility trends.

Introduction

MAP 4.1 Relative intergenerational mobility of income across the world

a. IGM for selected economies for which estimates are available

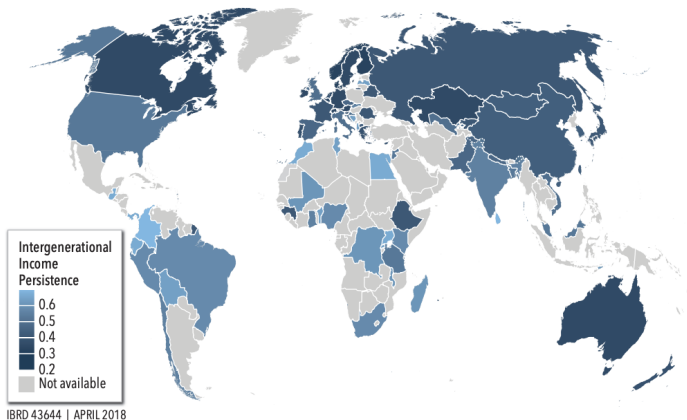
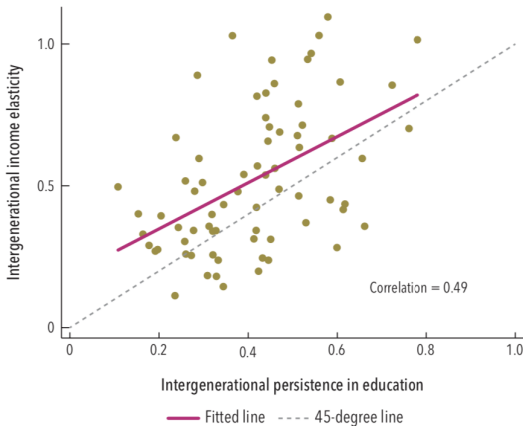


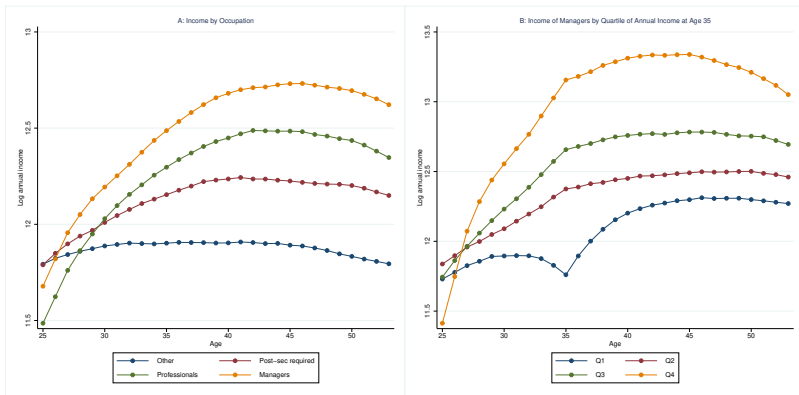
Figure: IGE Around the World

Relationship between mobility in income and education

FIGURE 1.2 Relative IGM in education and income are correlated, but imperfectly

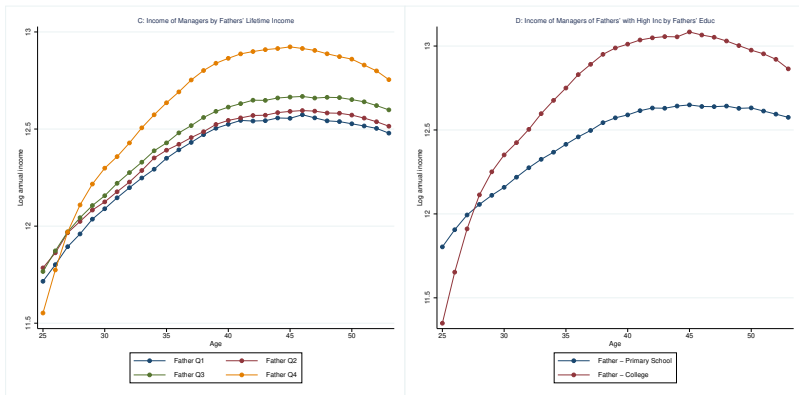


Evidence Swedish Data



[Back](#)

Evidence Swedish Data

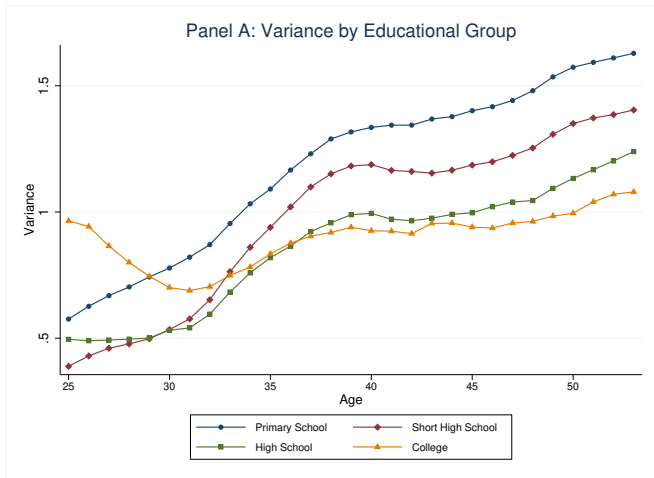


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Correction Methods

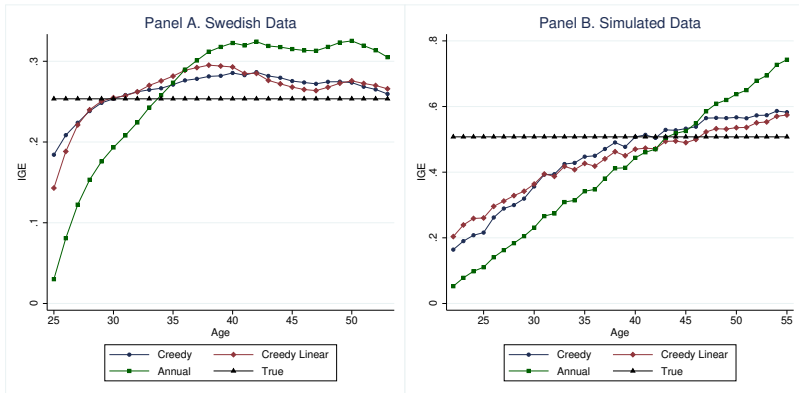
Modelling the Income Process - Creedy

Even controlling by education, variance of income increases with age.



Correction Methods

Modelling the Income Process - Creedy



Lifecycle Estimator

Figure: Comparison between Actual and Predicted Profiles

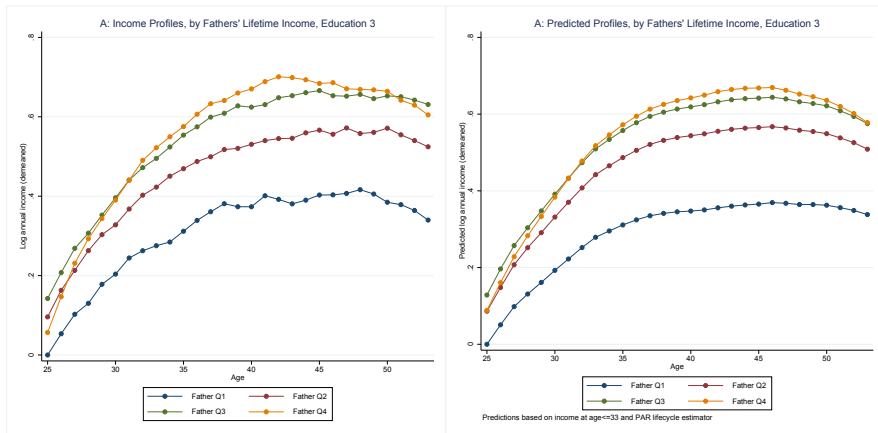


Table: The Lifecycle Estimator with Few Income Observations

Son's Age	N	Lifecycle estimator (Parental, Quadratic)				
		6 obs.	5 obs.	4 obs.	3 obs.	2 obs.
Age 30	187250	0.283*** (0.002) R2=0.076	0.289*** (0.002) R2=0.077	0.290*** (0.002) R2=0.076	0.284*** (0.002) R2=0.069	0.279*** (0.003) R2=0.062
Age 35	185608	0.263*** (0.002) R2=0.069	0.258*** (0.002) R2=0.064	0.257*** (0.002) R2=0.061	0.252*** (0.002) R2=0.056	0.256*** (0.003) R2=0.051
Son's Age	N	Lifecycle estimator (2-step, Quadratic)				
		6 obs.	5 obs.	4 obs.	3 obs.	2 obs.
Age 30	187250	0.240*** (0.003) R2=0.032	0.242*** (0.003) R2=0.032	0.236*** (0.003) R2=0.03	0.235*** (0.003) R2=0.03	0.238*** (0.003) R2=0.029
Age 35	185608	0.244*** (0.003) R2=0.041	0.242*** (0.003) R2=0.039	0.239*** (0.003) R2=0.037	0.237*** (0.003) R2=0.035	0.239*** (0.003) 0.032

Table: Varying the Sample Size

Sample size	1/4 N=46,356	1/16 N=11,624	1/64 N=2,936	1/256 N=732
Benchmark based on lifetime income				
Son's Age 25-53	0.260*** (0.010)	0.266*** (0.014)	0.259*** (0.037)	0.259*** (0.070)
Lifecycle estimator (Parental, Quadratic)				
Son's Age 25-30	0.277*** (0.007)	0.277*** (0.027)	0.277*** (0.050)	0.283*** (0.097)
Son's Age 25-35	0.259*** (0.015)	0.265*** (0.014)	0.262*** (0.040)	0.255*** (0.083)
Lifecycle estimator (2-step, Quadratic)				
Son's Age 25-30	0.255*** (0.011)	0.255*** (0.022)	0.243*** (0.047)	0.257*** (0.099)
Son's Age 25-35	0.246*** (0.008)	0.255*** (0.021)	0.251*** (0.040)	0.241*** (0.078)

Table 7: The Lifecycle Estimator in the PSID

Son's Age	Direct estimator		Lifecycle estimator				
	Lifetime	Annual	Baseline	Parental (Linear)	Parental (Quadratic)	2-Step (Linear)	2-Step (Quadratic)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Age ≤ 27	0.446*** (0.031)	0.270*** (0.016)	0.388*** (0.032)	0.407*** (0.032)	0.439*** (0.032)	0.456*** (0.039)	0.463*** (0.040)
N	1247	5947	1127	1127	1127	1127	1127
R ²	0.143	0.047	0.114	0.125	0.143	0.108	0.104
Age ≤ 30	0.446*** (0.031)	0.311*** (0.013)	0.406*** (0.032)	0.422*** (0.031)	0.450*** (0.031)	0.472*** (0.038)	0.477*** (0.038)
N	1247	9019	1159	1159	1159	1159	1159
R ²	0.143	0.060	0.126	0.135	0.151	0.120	0.118
Age ≤ 35	0.446*** (0.031)	0.354*** (0.011)	0.420*** (0.031)	0.430*** (0.030)	0.441*** (0.030)	0.480*** (0.036)	0.481*** (0.037)
N	1247	13900	1191	1191	1191	1191	1191
R ²	0.143	0.073	0.137	0.144	0.151	0.130	0.127
Age ≤ 40	0.446*** (0.031)	0.376*** (0.010)	0.430*** (0.032)	0.453*** (0.031)	0.451*** (0.031)	0.483*** (0.036)	0.482*** (0.036)
N	1247	18017	1229	1229	1229	1229	1229
R ²	0.143	0.076	0.131	0.145	0.144	0.126	0.125
Age ≤ 45	0.446*** (0.031)	0.385*** (0.009)	0.427*** (0.032)	0.440*** (0.031)	0.427*** (0.032)	0.463*** (0.035)	0.462*** (0.035)
N	1247	20603	1236	1236	1236	1236	1236
R ²	0.143	0.077	0.130	0.137	0.130	0.124	0.125

Figure: Income Profiles by Education Group and Cohort

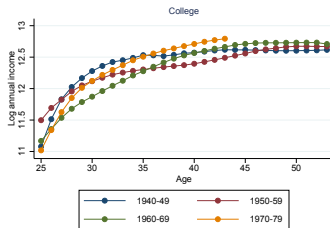
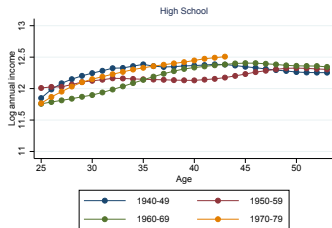
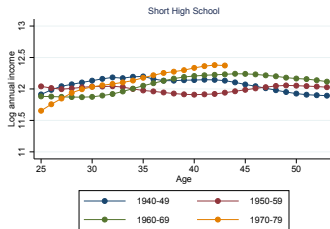
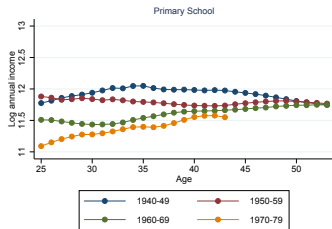


Table: Trends in Income Mobility in Sweden (Register data)

	Direct Estimator		Lifecycle estimator			
	Annual All ages	Annual Ages 25-30	Baseline	Parental (Quadratic)	Parental (Quadratic) Cohort Adj. 1	Parental (Quadratic) Cohort Adj. 2
	(1)	(2)	(3)	(4)	(5)	(6)
1950-59	0.253*** (0.003)	0.097*** (0.003)	0.219*** (0.003)	0.219*** (0.003)	0.219*** (0.003)	0.219*** (0.003)
1960-69	0.276*** (0.003)	0.180*** (0.003)	0.267*** (0.003)	0.280*** (0.003)	0.280*** (0.003)	0.280*** (0.003)
1970-79	0.273*** (0.003)	0.238*** (0.003)	0.281*** (0.003)	0.301*** (0.003)	0.304*** (0.003)	0.304*** (0.003)
1980-84	0.244*** (0.004)	0.235*** (0.004)	0.275*** (0.005)	0.303*** (0.004)	0.311*** (0.005)	0.308*** (0.005)
R2	0.028	0.021	0.051	0.056	0.059	0.059
N	15,136,452	4,112,196	775,972	775,972	775,972	775,838

Table: Trends in Income Mobility in the U.S. (PSID)

	Direct Estimator		Lifecycle estimator			
	Annual All ages	Annual Ages 25-30	Baseline	Parental (Quadratic)	Parental (Quadratic) Cohort Adj. 1	Parental (Quadratic) Cohort Adj. 2
	(1)	(2)	(3)	(4)	(5)	(6)
1950-59	0.396*** (0.042)	0.325*** (0.046)	0.431*** (0.048)	0.431*** (0.047)	0.430*** (0.048)	0.429*** (0.047)
1960-69	0.335*** (0.050)	0.304*** (0.046)	0.416*** (0.050)	0.418*** (0.050)	0.416*** (0.050)	0.420*** (0.050)
1970-79	0.353*** (0.038)	0.283*** (0.043)	0.392*** (0.044)	0.386*** (0.045)	0.394*** (0.045)	0.393*** (0.045)
1980-84	0.257*** (0.043)	0.271*** (0.052)	0.357*** (0.050)	0.376*** (0.050)	0.385*** (0.051)	0.388*** (0.051)
R2	0.078	0.073	0.141	0.141	0.145	0.145
N	28,241	8,149	2,238	2,238	2,238	2,238

Correction Methods

Modelling Errors-in-Variables

Generalized Errors-in-Variables (Haider & Solon, 2006)

- There is an age at which the expected difference between individuals' log annual incomes equals the expected difference between their log lifetime income $\rightarrow \lambda_t = 1$

$$y_{s,it} = \lambda_{s,t} y_{s,i}^* + u_{s,it},$$

- **Problem:** age at which $\lambda_{s,t} \approx 1$ is not known and it may vary substantially in a short window. GEiV

Our proposal: Standardized Errors-in-Variables

- Relates GEiV to moments that are more directly obtained. SEiV
 - Ratio between variance of annual and lifetime income.
 - Reliability ratio at age when incomes are observed.
- Method produces estimates within 5-10% of true IGE (still with fairly idealized conditions). Graph

Correction Methods

Modelling Errors-in-Variables

Table: Life-Cycle Bias and the Generalized-Errors-in-Variables Model

Swedish data			Simulated data		
Son's Age	$\lambda_{s,t}$	$\beta_{s,t}$	Son's Age	$\lambda_{s,t}$	$\beta_{s,t}$
31	0.810	0.208	41	0.896	0.461
32	0.869	0.224	42	0.958	0.470
33	0.940	0.243	43	0.997	0.506
34	1.007	0.258	44	1.036	0.518
35	1.072	0.273	45	1.047	0.525

Correction Methods

Own Correction Method - SEiV

Rewrite annual incomes as

$$y_{s,t} = \delta_{s,t} (y_s^* + u_{s,t}), \quad (4)$$

where $\delta_{s,t}$ is a scaling factor that may vary with age t .

Under this model, the slope in a regression of (log) annual income for sons $y_{s,t}$ on lifetime income of fathers y_f^* identifies:

$$\text{plim} \hat{\beta}_t = \frac{\text{Cov}(y_{s,t}, y_f^*)}{\text{Var}(y_f^*)} = \beta \delta_{s,t}, \quad (5)$$

However, individual-level data containing both annual and lifetime incomes are rarely available.

Correction Methods

Own Correction Method - SEiV

Note that the ratio between the variance of annual and lifetime incomes can be expressed as:

$$\frac{Var(y_s^*)}{Var(y_{s,t})} = \frac{Var(y_s^*)}{Var(\delta_{s,t}(y_s^* + u_{s,t}))} = \frac{1}{\delta_{s,t}^2} \frac{Var(y_s^*)}{Var(y_s^*) + Var(u_{s,t})} \quad (6)$$

which in turn implies that

$$\delta_{s,t} = \left(\frac{Var(y_{s,t})}{Var(y_s^*)} \right)^{\frac{1}{2}} \left(\frac{Var(y_s^*)}{Var(y_s^*) + Var(u_{s,t})} \right)^{\frac{1}{2}} \quad (7)$$

We can then replace sons' annual incomes $y_{s,t}$ by:

$$y_{s,t}^{std} = y_{s,t} \left(\frac{Var(y_{s,t})}{Var(y_s^*)} \right)^{-\frac{1}{2}} \left(\frac{Var(y_s^*)}{Var(y_s^*) + Var(u_{s,t})} \right)^{-\frac{1}{2}}$$

Correction Methods

Own Correction Method - SEiV

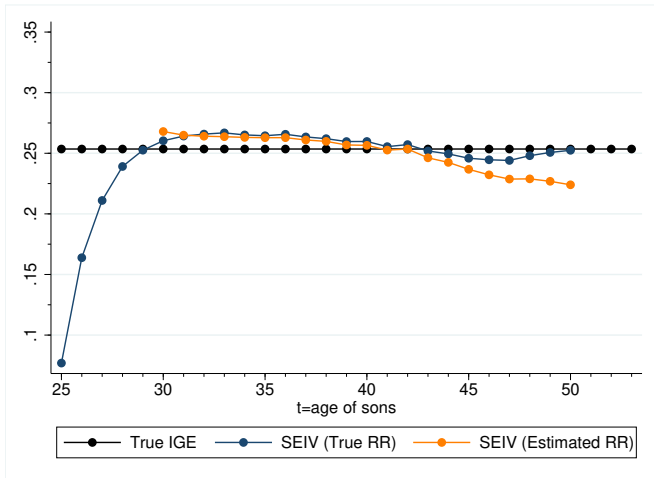


Figure: A standardized errors-in-variables model in Swedish Data